CHAPTER FOUR

QUADRATIC EQUATION

Introduction:

A quadratic equation is an equation which is of the form $ax^2 + bx + c = 0$.

Examples are:

1. $2x^2 + 4x + 1 = 0$
2. $5x^2 - 4x - 2 = 0$
3. $x^2 - x + 6 = 0$

For the determination of the truth set, the roots or the solution set of a quadratic equation, two methods can be used, and these are:

(i) By completing the squares
(ii) By using the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

QUESTION 1: Find the roots or the truth set of the quadratic equation $x^2 + 5x + 6 = 0$

SOLUTION

METHOD 1

$x^2 + 5x + 6 = 0$

=> ( ) ( ) = 0

=> (x) (x) = 0

We next consider all the factors of 6 and fix each set of factors in turn into the above and in each case, multiply the contents of the two brackets.

If the answer we get is the same as the given equation, then we proceed or continue working and if not, then we must try the other sets of factors.

The set of factors of 6 are:

–1 and – 6

2 and 3

3 and 2

– 6 and – 1

Fixing of the first set of factors => i.e. (–1 and – 6) into (x) (x) = 0

=> (x – 1) (x – 6) = 0.

Multiplying them out

$\Rightarrow x^2 - 6x - x + 6 = 0$

$=> x^2 - 7x + 6 = 0$

Since $x^2 - 7x + 6 = 0$ is not the same as $x^2 + 5x + 6$, which is the given equation, then – 1 and – 6 will not work.

We then substitute the next set of factors i.e. 2 and 3 into (x) (x) = 0

=> (x + 2) (x + 3) = 0

Multiply this out =>

$x^2 + 3x + 2x + 6 = 0$

$=> x^2 + 5x + 6 = 0$

Since this is similar to the given equation, we proceed with the work.

From (x + 2) (x + 3) =

=> (x + 2) = 0 => $x = 0 - 2$

=> $x = -2$, or (x + 3) = 0

=> $x = 0 - 3 = -3$

The truth set or roots is {x: $x = -2$ or $x = -3$}

NB: This method which is a try and error method is known as completing the square method.105

METHOD 2

If $ax^2 + bx + c = 0$,

Then $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The given equation is $x^2 + 5x + 6 = 0$.

=> $1x^2 + 5x + 6 = 0$.

Comparing this with $ax^2 + bx + c = 0$.

=> a = 1, b = 5 and c = 6.

=> $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

=> $$x = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(6)}}{2(1)}$$

$$x = \frac{-5 \pm \sqrt{1}}{2}$$

Since$\sqrt{1}$ = 1, then

$$x = \frac{-5 \pm 1}{2}$$

(i) Considering the positive sign

=> $$x = \frac{-5 + 1}{2} = \frac{-4}{2} = -2.$$

(ii) Considering the negative sign

$\Rightarrow x = \frac{-5-1}{2} = \frac{-6}{2}$

$\Rightarrow x = -3.$

$\therefore$ The roots or the truth set is $\{x: x = -2 \text{ or } x = -3\}$.

QUESTION 2: Determine the truth set of the equation $x^2 + 6x + 8 = 0$.

SOLUTION

METHOD (1)

$x^2 + 6x + 8 = 0$

$(x \quad\quad) (x \quad\quad) = 0.$

The set of factors of 8 are

(1) 1 and 8
(2) –2 and –4
(3) 8 and 1
(4) – 4 and –2
(5) 2 and 4
(6) 4 and 2
(7) –1 and – 8
(8) – 8 and –1

Substitute 1 and 8 into $(x \quad\quad) (x \quad\quad) = 0$

$\Rightarrow (x+ 1) (x+ 8) = 0,$

$\Rightarrow x^2 + 8x + x + 8 = 0,$

$\Rightarrow x^2 + 9x + 8 = 0.$

Since $x^2 + 9x + 8 = 0$ is not the same as the given equation which is $x^2 + 6x + 8$, we try the next set of factors which are –2 and – 4,

=> $(x - 2)\ \ (x - 4) = 0$,

=> $x^2 - 4x - 2x + 8 = 0$,

=> $x^2 - 6x + 8 = 0$,

Which is also different from the given equation i.e.

$x^2 + 6x + 8 = 0$.

We therefore try 2 and 4,

=> $(x + 2)\ \ (x + 4) = 0$,

=> $x^2 + 4x + 2x + 8 = 0$,

=> $x^2 + 6x + 8 = 0$.

Since this is the same as the given equation

=> $(x + 2)\ \ (x + 4) = 0$,

=> $x + 2 = 0$ => $x = 0 - 2$,

=> $x = -2$, or $(x + 4) = 0$,

=> $x = 0 - 4$ => $x = -4$.

The roots or truth set is given by $\{x: x = -2 \text{ or } x = -4\}$.

METHOD (2)

$x^2 + 6x + 8 = 0$

=> $1x^2 + 6x + 8 = 0$.

Comparing this with $ax^2 + bx + c = 0$.

=> a = 1, b = 6 and c = 8.

From $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\Rightarrow x = \frac{-6 \pm \sqrt{6^2 - 4(1)(8)}}{2(1)}$$

$$\Rightarrow x = \frac{-6 \pm \sqrt{36 - 32}}{2}$$

$$\Rightarrow x = \frac{-6 \pm \sqrt{4}}{2}$$

$$\Rightarrow x = \frac{-6 \pm 2}{2}$$

(i) Considering the positive sign

$$\Rightarrow x = \frac{-6+2}{2} = \frac{-4}{2} = -2.$$

(ii) Considering the negative sign

$$\Rightarrow x = \frac{-6-2}{2} = \frac{-8}{2} = -4.$$

The truth set = $\{x: x = -2 \text{ or } x = -4\}$.

QUESTION 3: Determine the values of *x*, which satisfy the equation $2x^2 + 8x + 6 = 0$.

SOLUTION

METHOD (i)

Since 2 is attached to the x^2, then this 2 must be attached to one of the *x* within the brackets.

=> (2*x*) (*x*) = 0.

The factors of 6 are 2 and 3, 3 and 2, -2 and -3, -3 and -2, I and 6, 6 and 1, -1 and -6 as well as - 6 and -1. Now let us try the set 2 and 3 => (2*x* + 2) (*x* + 3) = 0,

=> $2x^2 + 6x + 2x + 6 = 0$,

=> $2x^2 + 8x + 6 = 0$,

Which is the same as the given equation i.e. $2x^2 + 8x + 6 = 0$.

∴ (2*x* + 2) (*x* + 3) = 0,

=> $2x + 2 = 0$ => $2x = 0 - 2 = -2$.

$\therefore 2x = -2 \Rightarrow x = \frac{-2}{2} = -1$,

or $(x + 3) = 0 \Rightarrow x = 0 - 3 = -3$.

The roots or the values of x which satisfy the given equation is given by $\{x: x = -3 \text{ or } x = -1\}$.

Method (2)

Comparing $2x^2 + 8x + 6 = 0$

with $ax^2 + bx + c = 0$,

=> a = 2, b = 8 and c = 6, and from $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow x = \frac{-8 \pm \sqrt{8^2 - 4(2)(6)}}{2(2)}$

$\Rightarrow x = \frac{-8 \pm \sqrt{64 - 48}}{4}$

$\Rightarrow x = \frac{-8 \pm \sqrt{16}}{4}$

$\Rightarrow x = \frac{-8 \pm 4}{4}$.

Considering the positive sign

$\Rightarrow x = \frac{-8 + 4}{4} = \frac{-4}{4} = -1$.

Considering the negative sign

$\Rightarrow x = \frac{-8 - 4}{4} = \frac{-12}{4}$

=> $x = -3$. The truth set = $\{x: x = -1 \text{ or } x = -3\}$.